Key

1. For all **natural numbers n**, let S(n) be the sum of the digits of n plus the number of digits of n. For instance, S(125) = 1 + 2 + 5 + 3 = 11. Note that the first digit of n, when reading from left to right, cannot be zero.

   a. Determine S(12408)

   b. Determine all numbers m such that S(m) = 4

   c. Determine whether or not there exists a natural number m such that S(m)   S(m + 1) > 50. Provide a clear justification for your answer.

Problem 1 Solution:

a. **S(12408) = 1 + 2 + 4 + 0 + 8 + 5 = 20**.

b. Consider 1-digit numbers, then m = 3. Among 2-digit numbers we need those with the sum of their digits equal to 2; so we have m = 11 and m = 20. Among 3-digit numbers we need those with the sum of their digits equal to 1, so we have m = 100. For numbers with 4 or greater than 4q0.023(ith)-ng>37.33 481.87 Tm1 0 0 rg1 0 0 RG[ )]TJETQq0.00000912 0 612 7

2. Suppose line J in the xy-

3. Suppose g(x) is the quadratic function g(x) = x^2 ax + b, where a and b are **natural numbers.**

a. If a = b = 2, find the set of real roots of the expression g(x) x.

b. If a = b = 2, find the set of real roots of the expression g(g(x)) x.

c. Find the number of pairs of **natural numbers** 22, 22, and every root of the expression g(g(x)) x is an integer.

Problem 3 Solution

a. If a = 2 and b = 2, then g(x) = x^2 g = x^2
Therefore, the roots of g

b. We now determine g(g Note that g(g(x)) = (x^2 2 2 x^4
3

Since 1 22

^{2} 22 ^{2} 22 ^{2} 23. $\sqrt{2023}$

= 44, where t denotes the largest integer less than or equal to t. There are 43 solutions

4. Jaden takes a mathematics test consisting of 100 questions, where the answer to each question is either TRUE or FALSE. For every five consecutive questions on the test, the answers to exactly three of the questions are TRUE. If the answers to Question 1 and Question 100 are both FALSE:

a.

5. Suppose quadrilateral STRV is an isosceles trapezoid, with ST = 5 cm, RV = 5 cm, TR = 2 cm, and SV = 8 cm.

a. What is the the length of diagonal SR?

b. For the isosceles trapezoid in part (a), what is the exact value of the **cosine** of RTS?

c. In triangle KLM below, points G and E are points on segment LM so that MKG GKE EKL. Also, point F is located on segment KL so that segment GF is parallel to segment KM. If quadrilateral KFEG is an isosceles trapezoid and the measure of KLM is 84°, find the measure of MKG.

Problem 5 Solution:

a. Let TE be the altitude of the trapezoid, so that angle TES is a right triangle with hypotenuse ST = 5 and SE= $\frac{8-2}{2}$ = 3. Therefore, the altitude is 4 (the sides of triangle TES form the Pythagorean triple 3-4-5).

Using the fact that the altitude of the trapezoid is 4, construct altitude SF from point S. **From the right triangle SFR, where SF = 4, FR = FT + TR = 3 + 2 = 5, we find SR =**
√

b. Using triangle STR and the Law of

RTS.

So,

cos RTS 0)

As an alternative approach, since FT = SE = 3, and ST=5, cos FTS = 3/5.

Then cos RTS 0 + FTS) =

F

6. If M is a **natural number**