

**2013 John O'Bryan Mathematical Competition  
Junior-Senior Individual Test**

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. If  $x^4 - x^2 - 20 = 0$ , how many of the distinct roots are non-real?
2. Let  $i = \sqrt{-1}$ . If  $(3i)(ki) = -12$ , find the value of  $k$ .
3. Let  $k$  be an integer such that  $30 < k < 45$ . Find the sum of all distinct values of  $k$  such that  $k$  is **not** the sum of 2 or more consecutive **odd** natural numbers.
4. Find the sum of all distinct positive integer multiples of 7 if each multiple of 7 is less than 2013.
5. If  $x < 0$ , then find the value of  $x$  such that  $|2 - 3x| = 23$ .
6. One circle has an equation of  $(x - 8)^2 + (y + 2)^2 = 25$ . A second circle has an equation of  $2x^2 + 2y^2 + 12y = 1$ . Find the slope of a line that is perpendicular to the line that joins the centers of the two given circles.
7. The circumference of a circle is  $38.74\pi x$ . The area of this same circle is  $kx^2$ . Find the value of  $k$  rounded to the nearest integer.
8. An observer in a building that rises vertically notes that two objects on a horizontal road below have respective angles of depression of  $26^\circ$  and  $19^\circ$  respectively. If the distance from the base of the building to the first object is 100 feet and the second object is 150 feet from the base of the building, find the distance between the two objects.

10. It is known that 475 logs are piled in such a way that each row has one more log than the row above. If the

11. Let  $e$  be the base for natural logarithms and let  $\ln$  be the symbol for natural logarithms. Find, written as a single fraction in terms of  $e$ , the value of  $x$  such that  $\ln(x) - \ln(x-1) = 1$ .

12. Find the smallest positive integer that is an integral multiple of 18 distinct positive integers and is 4 less than the square of a positive integer.

Name: \_\_\_\_\_ ANSWERS \_\_\_\_\_

Team Code: \_\_\_\_\_

**Junior-Senior Individual Test**

**Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless**