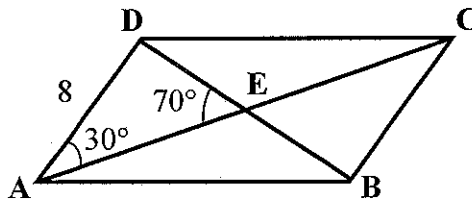


**2012 John O'Bryan Mathematical Competition**  
**Junior-Senior Individual Test**

**Directions:** Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

- Let  $i = \sqrt{-1}$ . Then  $i^7 - i^{13} = ki$ , where  $k$  is a real number. Find the value of  $k$ .
- Find the sum of the first thirty terms of the arithmetic sequence: 1.23, 2.37, 3.51, ... Express your answer as a decimal.
- In the diagram,  $ABCD$  is a parallelogram in which the diagonals intersect at  $E$ . If  $\overline{AD} = 8$ ,  $\angle DAC = 30^\circ$  and  $\angle DEA = 70^\circ$ , find the length of diagonal  $\overline{BD}$ . Express your answer as a **decimal** rounded to the nearest hundredth.



- A right circular cylinder has a **total** surface area of  $132\pi$ . Find the maximum volume of such a right circular cylinder. Express your answer as a **decimal** rounded to the nearest hundredth.
- If the magnitude of the three dimensional vector  $(2, 3, p)$  is  $\sqrt{38}$ , find the smallest possible value of  $p$ .
- Let  $x$  be an integer such that  $0 < x < 150$ . Find the sum of all possible distinct values of  $x$  such that  $\cos(2x + 8)^\circ > 0$  and  $\sin(5x - 12)^\circ < 0$ .
- A trapezoid has sides with respective lengths: 2, 41, 20, 41. Find the length of an altitude of this trapezoid.
- Rounded to the nearest centimeter, find the circumference of a circle in which a chord whose length is 80 centimeters is 9 centimeters from the center of the circle.
- When two boys mow a lawn, one works at twice the rate of the other. If it takes the two boys a total of 3 hours to mow the lawn when they work together, how many hours would it take the faster boy to mow the lawn by himself?
- A triangle has vertices at  $(0,0)$ ,  $(5,6)$ , and  $(5,-2)$ . A point is selected at random in the interior of the triangle. Find the probability that the point selected lies in Quadrant I. Express your answer as a common

11. Solve the determinant equation for  $k$ :  $\begin{vmatrix} 1 & -2 & -6 \\ 4 & 1 & 0 \\ 5 & -3 & k \end{vmatrix} = 165$ .

12. Let the equation of a parabola be  $y = x^2 - kx + w$ . The sum of the squares of the  $x$ -intercepts of the

Name: \_\_\_\_\_ **ANSWERS** \_\_\_\_\_

Team Code: \_\_\_\_\_

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1.           -2          

11.           7          

2.           532.8          

Must be this decimal

12.           -1429          

Must be this reduced improper

4.           648.36          

Must be this decimal, cubic units optional

14.           50          

5.           -5          

15.           1078          

6.           2167          

16.           108,864          

7.           40          

17.           148